Derivation Supplement

Origin of the four quantities used in the Letter, and derivation of the bump

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Purpose. This supplement derives the quantities the Letter uses — the presence fraction f = 1/16, the threshold $r_t = \sqrt{(2A)}$, the extinction factor and its 2π, the many-body superposition rule, and the bump above the threshold — from a single postulate: a tension interface whose static equilibrium is $\sigma \cdot \mathrm{Tr}(K) = \rho_E$. It presents no other part of the underlying framework. Each quantity is obtained with no fitted parameter; none is calibrated on binary data.

1. The presence fraction f = 1/16

What the presence fraction is. The source does not contribute to the interface Σ permanently: in each field, it spends part of its phase on Σ — present — and part outside Σ — absent. The presence fraction f measures the proportion of phase during which it is effectively present on Σ.

Where the value 1/16 comes from. The problem comprises four structures: the three fields and the interface that separates them. In each, the source is at times present, at times outside; averaged over phases, it is present half the time — a factor ½ per structure. The four being independent, the source is effectively present on the interface only when the four coincide, and independent states compose as a product:

$$f = (1/2)^4 = 1/16 \,.$$

This is a counting property, not an adjustment. The number 1/16 has no degree of freedom: it is fixed once one counts four structures each present half the time.

2. The threshold r_t = √(2A)

The transition separates two regimes according to the comparison between the variation of the perturbation and the curvature of the background. Above the threshold, the gradient of the perturbation varies fast against the background curvature: the latter remains invisible and the dynamics is Newtonian. Below, the perturbation varies more slowly than the background curves, and the expansion around a flat background loses its meaning — this is where the dynamics deviates.

The radius where the gradient of the metric perturbation crosses the background curvature — the crossing radius — is written with the framework's quantities alone, the object's energy E and the tension σ:

$$r_t = \sqrt{(2A)} = \sqrt{(E/(6\pi\sigma))} \,,$$

where $A = E/(12\pi\sigma)$ is the shape constant of the source's proper field. No parameter is adjusted. The threshold is written with the object's quantities alone and applies, with the same coefficient, at any scale where a tension source meets the background; for a stellar pair, it falls at several thousand astronomical units.

Mass dependence. Since E grows with the object's mass, the threshold grows as √M. It is therefore not universal: the position of the step shifts from one pair to another according to its mass (Figure 2 of the Letter).

Acceleration at the threshold. The Newtonian acceleration evaluated at the crossing radius is independent of the source: it equals $12\pi G\sigma/c^2$. This is the unique acceleration scale at which the transition operates, whatever the mass. Since the tension σ that fixes it is inherited from the vacuum energy density, and not from an acceleration scale, its coincidence with the observed threshold of the binaries is a consequence of the cosmological value of ρ_Λ, not a calibration.

Origin of σ. The interface tension is fixed by two independently measured quantities, the vacuum energy density ρ_Λ and Newton's constant G, by inversion of the relation $G = c^4\rho_\Lambda/(12\pi\sigma^2)$: $\sigma = \sqrt{(c^4\rho_\Lambda/12\pi G)} = 4.12\times10^{16}\ \mathrm{J\cdot m^{-2}}$. No acceleration scale enters this determination. The closure curvature $k = \rho_\Lambda/\sigma$ is a derived quantity. The value of σ is therefore independent of the wide-binary data.

3. The extinction factor and its 2π

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Above the threshold, the crossing coordinate ceases to oscillate freely and becomes evanescent. Its behaviour is governed by the phase equation of the mode whose effective potential is the threshold itself: below the threshold, the mode is oscillatory (phase equipartition is recovered, and gives the 1/16 of Section 1); above, it is evanescent.

The phase equation. Writing φ for the crossing angle and ψ for the mode amplitude, the mode obeys a phase equation whose effective potential is the threshold itself,

$$d^2\psi/d\varphi^2 = (x - 1)\cdot\psi \,,$$

with two regimes across $x = 1$. For $x < 1$ the right-hand side is negative and the mode is oscillatory, $\psi \sim \cos(\sqrt{(1-x)}\cdot\varphi)$, $\sin(\sqrt{(1-x)}\cdot\varphi)$ — phase equipartition, which yields the 1/16 of Section 1. For $x > 1$ the right-hand side is positive and the mode is evanescent, $\psi \sim e^{-\sqrt{(x-1)}\cdot\varphi}$. The crossing coordinate φ is an angle of period 2π (one full turn), so the presence amplitude over a complete turn decays as $e^{-2\pi\sqrt{(x-1)}}$: the extinction factor and its 2π are read directly from the mode, not posited.

Origin of the 2π. The crossing coordinate is an angle, whose natural period is one full turn. The factor 2π appearing in the exponent is that period — the angle closing upon itself. The absence fraction above the threshold thus decreases as

$$1 - f_1 = \tfrac{1}{2}\cdot e^{-2\pi\sqrt{(x-1)}} \,,$$

continuous at the threshold $x = 1$. The 2π is not adjusted: it is the period of the crossing angle, imposed by the geometry of the mode.

The coefficient 0.323. The additional slope below the threshold is fixed by a chain — amplitude interference, scale tensions, shape equation — whose result, in this sector of the framework, reads $(2\cdot 2/3)\cdot\sqrt{(f(1-f))}\cdot\sqrt{A/r}$. The factor $2\cdot(2/3) = 4/3$ combines a factor 2 from the interference of the two amplitudes and the 2/3 of the shape equation (the missing third in spherical geometry); it contains no free parameter. The dynamical ratio then reads

$$g_{obs}/g_N = 1 + (4/3)\cdot\sqrt{(f(1-f))}/\sqrt{x} = 1 + 0.323/\sqrt{x} \,,$$

the number 0.323 being $(4/3)\cdot\sqrt{(f(1-f))}$ evaluated at $f = 1/16$. Once the three fields and the interface are counted, and the factor 4/3 fixed by the chain above, the slope below the threshold has no degree of freedom: the comparison of this value with binary data is a test, not a fit.

4. Many-body superposition

The effect of a local field on a stellar pair requires composing two sources. The framework does this exactly, by a structural property of its field equation.

Additive displacements. The field equation $\sigma\cdot \mathrm{Tr}(K) = \rho_E$ is linear in the displacement u: if two sources each produce a solution, their sum is a solution. The displacements add, $u_1 + u_2$.

Multiplicative clock rates. But the clock rate — the temporal factor that carries the dynamical effect — is exponential in that same displacement. For two bodies:

$$e^{2k(u_1+u_2)} = e^{2k\cdot u_1}\cdot e^{2k\cdot u_2} \,.$$

The clock rates multiply, exactly as they do between layers of a single well. The composition principle that fixes the second order thus extends to the many-body problem, exactly solvable at the field level, with no external-field-effect parameter. In other words, in this framework gravity does not gravitate: all the nonlinearity resides in the dressing of the geometry, none in the field equation.

Consequence — the plateau. For a pair immersed in a background field, the background displacement u_{loc} adds to that of the pair before entering the exponential; at large separation, where the pair's proper field fades, only the background clock rate survives. The ratio then tends toward a plateau fixed by the local baryonic slope alone $x_{loc} = g_{bar}\cdot c^2/(6\pi G\sigma)$:

$$\gamma(s\to\infty) = 1 + 0.323/\sqrt{x_{loc}} \,.$$

The height of the plateau is therefore not a parameter: it is fixed by the numerical value of the local baryonic slope g_{bar}, which published models place in the interval $g_{bar} \approx (1.5\text{–}1.9)\times 10^{-10}\ \mathrm{m\cdot s^{-2}}$, whence $\gamma(\infty) \approx 1.56\text{–}1.63$. Its derivation

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contains no quantity fitted on the binaries. The dependence of the plateau on the local value of the background field is a prediction: the plateau rises where the local baryonic slope is weaker.

5. The bump above the threshold: derivation and projection

The Letter notes a fine bump of the ratio just above the threshold. This bump admits a closed derivation, with no parameter, from the same chain as the slope below the threshold; its peak height is an exact fraction, and its survival under projection onto the observables is verified by the same Monte Carlo draw as the figures.

The unified form. The coefficient (4/3) and the interference term $\sqrt{f(1-f)}$ of Section 3 do not depend on the regime; only the presence fraction f changes. The ratio of the two scale tensions — that of the fold, $\sigma(2u/r^2)$, and that of the vacuum, $\sigma(2/r)$ — equals exactly the Newtonian slope, $\sigma(2u/r^2)/\sigma(2/r) = u/r = x$: the variable x is that internal ratio. Below the threshold, the vacuum dominates and equipartition fixes $f = 1/16$; above, the fold occupies the front and the crossing coordinate becomes evanescent, so that the presence rises according to

$$f(x) = 1 - \tfrac{1}{2} \cdot e^{-2\pi\sqrt{x-1}} \quad (x \geq 1), \text{ continuous at the threshold where } f = \tfrac{1}{2}.$$

The dynamical ratio thus reads, on both sides of the threshold, in a single form,

$$g_{obs}/g_N = 1 + (4/3) \cdot \sqrt{f(1-f)}/\sqrt{x},$$

with $f = 1/16$ below the threshold and $f = f(x)$ above. There is no new ingredient: the same chain governs both regimes.

The peak height, exact. At the threshold, the presence crosses its maximal interference value $f = \tfrac{1}{2}$: the term $\sqrt{f(1-f)}$ reaches there its maximum ½. The ratio therefore peaks at

$$\gamma_{peak} = 1 + (4/3) \cdot (\tfrac{1}{2}) = 1 + 2/3 = 1.667 \text{ (exactly)},$$

with no degree of freedom — imposed by $f = \tfrac{1}{2}$ and the coefficient 4/3 already fixed below the threshold. The peak then descends toward the plateau as the presence saturates ($f \to 1$) and the interference dies out: $\gamma \approx 1.43$ at $x = 1.05$, ~1.32 at $x = 1.10$. The bump is therefore narrow and tall, centred on the threshold.

Survival under projection. It remains to know whether this fine structure survives the passage into the observable plane. We project it by the same Monte Carlo draw as the figures of the Letter: $N = 6\times10^5$ systems with log-uniform semi-major axes over [0.8; 40] kAU (kilo-astronomical units, 10^3 AU), thermal eccentricities $p(e) = 2e$, sampled orbital phases (Kepler's equation), isotropic line of sight; the background-field effect is composed exactly, $x = \sqrt{x_{proper}^2 + x_{loc}^2 + 2 \cdot x_{proper} \cdot x_{loc} \cdot \mu}$, μ being the cosine of the angle between the two gradients. For a total mass of 1.5 M⊙ ($r_t \approx 3.9$ kAU), the projected ratio with the bump exceeds the smoothed ratio by

$$\Delta\gamma \approx +0.06 \text{ (2.5–3 kAU)}, +0.09 \text{ (3–3.5 kAU)}, +0.07 \text{ (3.5–4 kAU)},$$

the gap falling back to +0.02 by 4–5 kAU. The bump is therefore not erased by projection: it leaves an observable shoulder in the 3–4 kAU window, visible in Figure 1 of the Letter just before the rise toward the plateau. This signature — a narrow shoulder, indexed on the threshold r_t and hence shifted by the mass of the pair — constitutes an additional prediction, distinct from the plateau, and requires a sample finely resolving the threshold zone.

Summary

The quantities of the Letter all follow from the single postulate of a tension interface: $f = 1/16$ by counting (three fields, one interface); $r_t = \sqrt{2A}$ as the crossing radius of the perturbation gradient and the background curvature; the 2π as the period of the crossing angle in the phase equation of the mode; multiplicative superposition as a consequence of the linearity of the field equation in the displacement; and the bump above the threshold as the same interference chain, with $f = \tfrac{1}{2}$ at the threshold giving the exact peak $\gamma = 1 + 2/3$, whose shoulder at 3–4 kAU survives projection. None is calibrated on wide-binary data: the comparison of the Letter with Gaia is therefore a test, not a fit.

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