A transition function derived from a tension interface, with a sharp threshold: derivation and falsifiable signatures

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Abstract. Modified-gravity phenomenology at low acceleration is usually captured by interpolating functions introduced by hand. We show that a transition function follows instead from the static equilibrium of a tension interface, $\sigma \cdot Tr(K) = \rho_E$, derived from an action and containing no adjustable parameter. The resulting form differs qualitatively from the usual interpolations: it predicts a sharp threshold at a separation $r_t = \sqrt{(2A)}$, a transition two to three times steeper than standard interpolations, and a plateau whose height is fixed by the local baryonic field rather than being universal. The single dimensional input, the interface tension, is fixed by the cosmological vacuum energy density and Newton's constant, so that no acceleration scale is fitted. Confronted with Gaia wide binaries through a projection pipeline applied identically to all models, the framework separates from MOND-type interpolations by up to 0.28 in dynamical ratio in the 4–10 kAU (kilo-astronomical units, 10^3 AU) window, a zone accessible to current deep samples. We provide the discriminating curves and delineate where the prediction can be falsified; we do not arbitrate the ongoing observational controversy.

Context. Since 2023, several analyses of Gaia wide binaries have reported, at large separations, an excess relative velocity compared to the Newtonian prediction, in the regime where the internal acceleration falls below $\sim 10^{-10}$ m·s^{-2}: some groups read this as the signature of a departure from standard gravity [3,4,7,8], others conclude Newtonian compatibility [6,10]. The question remains open and debated. We do not enter this arbitration: we derive, from an independent theoretical framework, a transition function whose form is distinct from both Newton and MOND-type interpolations, and whose signatures are confrontable with the same data.

1. The foundation: a tension interface between three fields

The fundamental object of this framework is an interface Σ — a codimension-1 hypersurface — carrying a tension σ. The low-acceleration modification does not arise from an addition to the law of gravitation, but from the static equilibrium of this interface.

The action and its equilibrium. The action of an interface combining tension, coupling to energy density, and curvature rigidity reads

$$S[\Sigma] = \sigma \int_\Sigma dA - p \int_\Omega dV + (\kappa/2) \int_\Sigma (H - H_0)^2 \, dA,$$

where dA is the induced area element on Σ, dV the volume element of the enclosed domain Ω, H = Tr(K) the mean curvature (trace of the extrinsic curvature K), and (σ, p, κ, H_0) the couplings. Stationarity $\delta S = 0$ under any variation of the interface imposes the shape equation — the generalized Young-Laplace law, $\sigma \cdot Tr(K) = -p$. The conjugate "pressure" being here the energy density ρ_E that the interface separates, one obtains the exact static form

$$\sigma \cdot Tr(K) = \rho_E .$$

The equilibrium is therefore not postulated: it is the minimality condition of the action, its first variation. It possesses a complete dynamical form, linear in the displacement u,

$$(3\sigma/c^2)\, \partial^2_t u - 3\sigma\, \nabla^2 u + \kappa\, \nabla^4 u = -\rho_E ,$$

whose linearity is what allows, below, the multiplicative composition of clock rates for several bodies.

Regime of validity. The present construction sits in the static, weak-field regime. In this regime, the exponential clock rate carried by matter identifies with the temporal component of the metric, $g_{00} = e^{(2\Phi/c^2)}$, and the Letter restricts itself to the dynamics of bound systems at low acceleration. The treatment of light propagation — deflection, lensing — involves the response of the interface at higher order and is the subject of separate work; it lies outside the scope of this Letter.

Why three fields. The interface separates three fields. This number is not posited arbitrarily. That physical space is three-dimensional is strongly constrained by the stability of bound systems: Ehrenfest [1] showed that planetary orbits and the stationary states of electrons in atoms become unstable beyond three dimensions, a result extended by

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Tangherlini [2] to general relativity and to the quantum hydrogen atom. In the present framework, to each spatial dimension corresponds one field — the correspondence $N = d = 3$; the number of fields is therefore three. The factor 3 of the field equation, and the counting that fixes the presence fraction $f = 1/16$ (Supplement), follow directly.

2. The derived transition function

The dynamical ratio follows from the equilibrium $\sigma\cdot\mathrm{Tr}(K) = \rho_E$ with no fitting assumption. The control variable is the Newtonian slope $x = |\nabla u_N|$, dimensionless, whose threshold equals unity. Below the threshold, intermittency interference gives

$$g_{obs}/g_N = 1 + (4/3)\cdot\sqrt{f(1-f)}/\sqrt{x} = 1 + 0.323/\sqrt{x}\ ,\ \ f = 1/16\ ,$$

where $f = 1/16$ is the presence fraction, a structural property (four binary structures: three fields plus the interface). Above the threshold, the crossing coordinate becomes evanescent and the ratio is damped by an extinction factor

$$\tfrac{1}{2}\cdot e^{-2\pi\sqrt{x-1}}\ ,$$

whose factor 2π is the period of the crossing coordinate — an angle closing over one turn. None of these numbers is adjusted: the coefficient 0.323 is $(4/3)\cdot\sqrt{f(1-f)}$ evaluated at $f = (\tfrac{1}{2})^4 = 1/16$; the threshold $\sqrt{2A}$ arises from the radius where the source's proper field crosses the vacuum floor; the 2π comes from the phase equation of the crossing mode. The same quantities (the threshold, the 1/16, the 2π) govern, in this framework, the mass gap between the first two lepton generations: the transition function is not proper to binaries, but an instance of an intermittency structure.

The same field equation being linear in the displacement, the displacements of several sources add, and the clock rates (exponential in the displacement) multiply: $e^{2k(u_1+u_2)} = e^{2ku_1}\cdot e^{2ku_2}$. This superposition rule, exact, fixes the effect of a local field with no parameter.

The tension and its origin. All the numerical predictions below pass through the interface tension σ. This is not a free parameter nor fitted on the binaries: it is fixed by inversion of two independently measured quantities, the vacuum energy density $\rho_\Lambda \approx 5.3\times10^{-10}\ \mathrm{J\cdot m^{-3}}$ and Newton's constant G,

$$\sigma = \sqrt{c^4\rho_\Lambda/12\pi G} = 4.12\times10^{16}\ \mathrm{J\cdot m^{-2}}\ .$$

No acceleration scale enters this determination; the closure curvature $k = \rho_\Lambda/\sigma$ is a derived quantity. This is why the coincidence, noted in conclusion, between the Newtonian acceleration at the derived threshold, $6\pi G\sigma/c^2$, and the scale where the analyses locate the onset of the anomaly is not a tautology, but a consequence of the cosmological values of ρ_Λ and G.

This relation serves here only to fix the value of σ from two measured constants: it is a parametrization, not a claim. The physical interpretation of the link between G and ρ_Λ belongs to the complete framework and lies outside the scope of this Letter. No acceleration scale enters the determination of σ, and no quantity is fitted on the binary data.

Accordingly, the exact statement of the model's economy is: the transition function comprises four equations and no parameter fitted on wide-binary data — the only dimensional scale, σ, being inherited from the cosmological sector. The comparison of the following sections with Gaia is therefore a test, not a fit.

3. Three signatures, and the central figure

The figure below compares, for a total mass of 1.5 M⊙, the ratio $\gamma = g_{obs}/g_N$ predicted by the framework and by MOND with external-field effect (simple interpolating function, external field $g_{ext} = 1.8\ a_0$), both passed through the same projection machinery (projected separations and velocities, thermal eccentricity distribution, orbital phases), so as to be compared on equal footing in the observable plane. For reference, the published AQUAL predictions for wide binaries give a plateau $\gamma \approx 1.35$–1.40. Newtonian gravity (dotted) serves as reference.

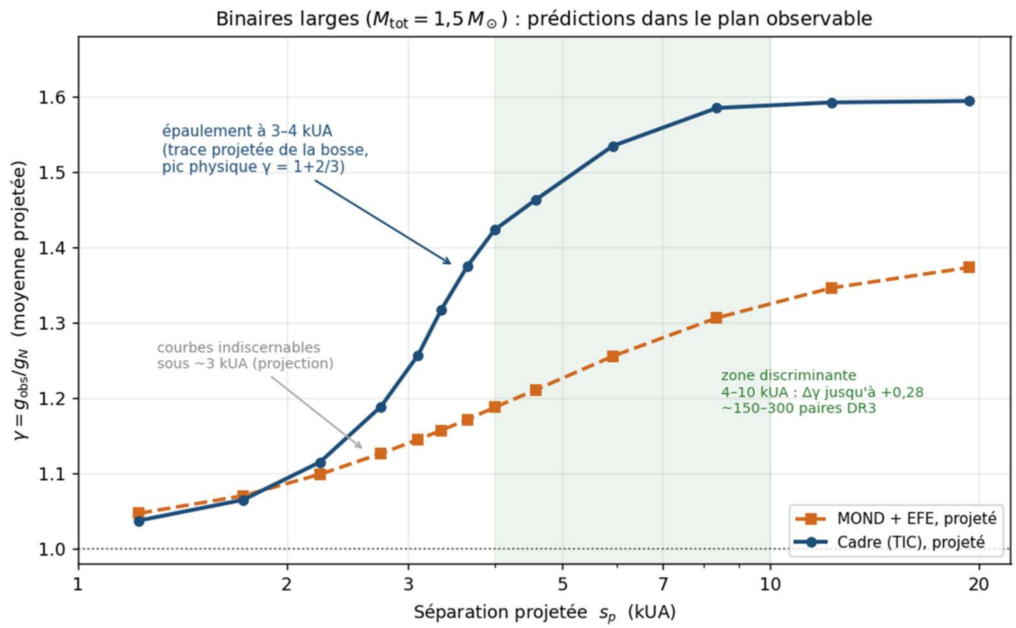

Figure 1. *Predictions in the observable plane (means projected over eccentricities and phases). Below ~3 kAU, projection renders the two models indistinguishable; between 4 and 10 kAU, the framework rises two to three times faster and reaches its plateau by ~7 kAU, while MOND keeps growing gradually — a gap reaching 0.28 in dynamical ratio in a zone accessible to Gaia DR3. The derived bump (physical peak γ = 1 + 2/3) leaves, after projection, a shoulder at 3–4 kAU, visible here just before the rise toward the plateau.*

The three signatures are all fully derived.

(1) The step. The threshold falls at $r_t = \sqrt{(2A)} = 0.456 \cdot r_{\mathrm{MOND}}$, and grows as $\sqrt{M}$ (2 266 AU at 0.5 M⊙, 5 068 AU at 2.5 M⊙; Figure 2). The $\sqrt{M}$ scaling is shared with MOND and therefore does not discriminate on its own; it is the coefficient 0.456 — which places the framework's step inside the MOND radius — and the abrupt character of the transition that distinguish the two predictions. Sorting the binaries by mass and measuring the position of their transition thus constitutes a test in its own right.

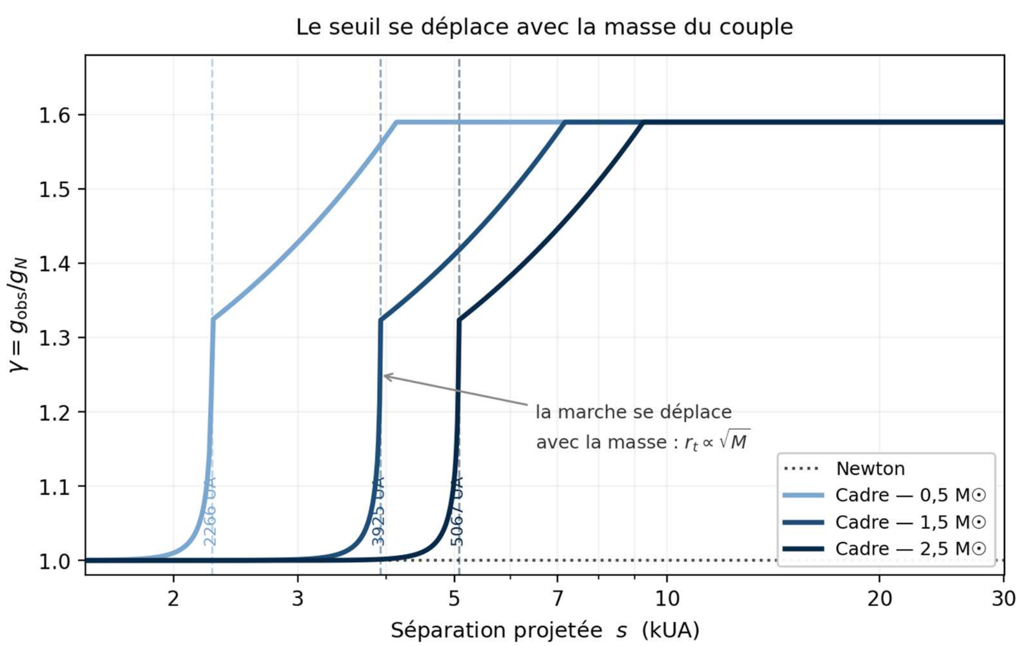

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Figure 2. The step shifts with the total mass of the pair ($r_t \propto \sqrt{M}$). *Sorting the binaries by mass and measuring the position of their transition constitutes a test in its own right, independent of the plateau height.*

(2) The steepness. In the physical plane, the dynamics remains Newtonian below the threshold then rises abruptly; projection onto the observables softens this step, but the rise remains two to three times steeper than that of the usual interpolations. Between 4 and 10 kAU projected, the ratio goes from 1.21 to 1.59, against 1.15 to 1.31 for MOND — a gap reaching 0.28, in the sense that the framework is higher.

(3) The plateau. The framework reaches its plateau, $\gamma \approx 1.56$–1.63, by about 7 kAU, while MOND keeps growing beyond 20 kAU without attaining it. The height of this plateau comes not from the framework but from a measurable, local input: the value of the baryonic slope g_{bar} at the location of the pair. The framework therefore predicts that the plateau is not universal — it rises where the background baryonic field is weaker. On the current sample, this input is nearly uniform and the plateau constant; its variation with the local environment is a testable prediction, distinct from the height itself.

Smoothing assumptions. The projected values above result from a draw of 6×10^5 systems with log-uniform semi-major axes, thermal eccentricities and sampled orbital phases, under isotropic line of sight. This is a first-order smoothing: it includes neither selection cuts, nor measurement noise, nor orbit integration under the modified force. A complete confrontation with data would require a statistical inference of the kind employed by recent analyses; it constitutes the next step and is not undertaken here.

The table gathers the projected values of Figure 1.

s_p (kAU)	**1–1.5**	**2–2.5**	**2.5–3**	**3–3.5**	**3.5–4**	**4–5**	**5–7**	**7–10**	**15–25**
Framework (TIC)	1.03	1.09	1.19	1.29	1.39	1.46	1.54	1.59	1.60
MOND + EFE	1.05	1.10	1.13	1.15	1.18	1.21	1.26	1.31	1.37
Newton	1	1	1	1	1	1	1	1	1

Table 1. $\gamma = g_{obs}/g_N$ averaged in the observable plane, for $M_{tot} = 1.5\ M\odot$, as a function of projected separation s_p. Local baryonic field g_{bar} *between 1.5 and 1.9 (in 10^{-10} m/s²).*

The framework predicts in addition a fine bump of the ratio just above the threshold. The same chain that fixes the slope below the threshold governs the regime above: only the presence fraction changes, passing from 1/16 (equipartition, below the threshold) to $f(x) = 1 - \frac{1}{2}\cdot e^{-2\pi\sqrt{x-1}}$ (evanescence, above), continuous at the threshold where it equals ½. The ratio thus reads, on both sides, in a single form: $g_{obs}/g_N = 1 + (4/3)\cdot\sqrt{f(1-f)}/\sqrt{x}$. At the threshold, the presence crosses its maximal interference value $f = \frac{1}{2}$, and the ratio peaks there at $\gamma = 1 + 2/3 = 1.667$ exactly, with no parameter; it then descends toward the plateau ($\gamma \approx 1.43$ at $x \approx 1.05$). In separation, this peak falls in the vicinity of the threshold r_t — for 1.5 M⊙, an observable shoulder around 3 to 4.5 kAU — which requires a sample finely resolving the threshold zone.

4. Testability

Immediate (Gaia DR3). The discriminant lies between 4 and 10 kAU projected, where the gap between the framework and MOND reaches 0.23 to 0.28 in dynamical ratio, i.e. a relative-velocity difference of 9 to 11 %. Given a per-pair dispersion of about 35 % (projection and eccentricity), of order 150 to 300 pairs suffice to separate the two models at 3–5 σ in this zone. By a clean pair we mean here a system passing the usual cuts of gravity tests — relative parallax better than 20 %, projected relative velocity compatible with a bound state, rejection of resolved companions and chance alignments; the required count is comparable to that of current deep Gaia DR3 samples. Below about 3 kAU, by contrast, projection renders the two models indistinguishable: this zone does not discriminate.

Forthcoming (Gaia DR4). The plateau height is cleanly measured with a few hundred pairs beyond 10 kAU; the fine bump requires a mass-resolved sample.

Solar-system limit. At small separations, the Newtonian slope x becomes very large and the extinction factor $e^{-2\pi\sqrt{x-1}}$ suppresses any deviation. Near the Sun, x is of order 10^5 at 10 AU and grows further inward: the extinction there is below 10^{-80}, so that the dynamics remains Newtonian to a precision far beyond any ephemeris constraint, including those of Cassini. The modified regime is strictly confined to very low accelerations.

5. Scope

We provide discriminating curves and the corresponding decision zones; we do not arbitrate the ongoing observational controversy over the existence of the anomaly. What the framework brings is a transition form — sharp threshold, step, plateau — specific and falsifiable, distinct from both Newtonian gravity and MOND-type interpolations. In particular, the plateau predicted here, $\gamma \approx 1.56$–1.63, sits above the asymptotic AQUAL value with external-field effect ($\gamma \approx 1.35$–1.40) and above recent empirical estimates ($\gamma \approx 1.4$–1.5): this height gap is in itself a discriminant. We note finally that the Newtonian acceleration at the threshold, $6\pi G\sigma/c^2 \approx 5.8\cdot 10^{-10}$ m/s², also reads $\sqrt{(3\pi G\rho_\Lambda)}$: it coincides with the scale where the analyses locate the onset of the anomaly ($\approx 5\times10^{-10}$ m/s²) and belongs to the family of constructions relating the transition acceleration scale to the cosmological vacuum, and the framework's own contribution is not this scale — common to those constructions — but the form of the transition: its position at $0.456\cdot r_{MOND}$, its abruptness, and a plateau whose height is indexed on the local baryonic slope.

We further note that this threshold acceleration should not be confused with the front acceleration, $12\pi G\sigma/c^2 \approx 1.2\times10^{-9}$ m/s², which is twice it: the former is the Newtonian value at the crossing radius $r_t(GM/(2A) = 6\pi G\sigma/c^2 \approx 5.8\times10^{-10}$ m/s²), where the dynamics deviates; the latter is a distinct derived quantity of the framework and does not enter the comparison with the observed threshold.

We remain deliberately in the context of Gaia wide binaries: this is the low-acceleration regime where data of sufficient precision exist to render the framework's predictions falsifiable. Extending the construction to other scales would be formally straightforward, but would move the discussion outside the domain where it can be refuted; we prefer to state signatures first where they can be confronted.

Finally, the transition function established here is not proper to binaries: the same construction, applied to other scales where a tension source meets the background, extends to the low-acceleration regime in general. The consequences of this extension will be the subject of separate work.

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